

Quark Coalescence and Hadronic Equilibrium

T. S. BIRÓ*

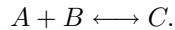
CERN THEORY DIVISION, CH-1211 GENEVE 23, SWITZERLAND

Abstract In a simplified model we study how close can the result of a fast non-equilibrium process, like hadronization of quark matter in an expanding fireball, come to hadronic composition in equilibrium. In a chemical approach of in-medium quark fusion ($A + B \rightarrow C$) we find that this depends on the time-integrated rate per volume multiplied by the conserved number of constituents. Ideal hadron gas equilibrium constrained by conservation laws, the fugacity parametrization, as well as linear and non-linear coalescence approaches are recognized as different approximations to this chemistry. It is shown that color confinement requires a dependence of the hadronization cross section on quark density.

Since strangeness enhancement had been suggested as a possible signature of quark-gluon plasma formation in heavy-ion collisions, there is a particular attention to the question of hadronic and quark level equilibrium in these reactions [1–4]. Recent claims of describing data at AGS as well as at CERN SPS by constrained hadronic equilibrium (i.e. using a common volume, temperature and only chemical potentials of conserved quantities for the computation of hadron numbers) “quite” satisfactorily [5–9] led to some astonishment in the community: the total reaction time at SPS energy is namely too short for achieving hadronic equilibrium by hadronic conversion processes alone. Pushing aside the faux pas, that this “satisfactory” equilibrium description agrees with experimental data for a few ratios only within a factor of two while the general agreement is on the 30% level, one faces the question, how can this simple idea work even so well. The time argument above would exclude this.

Meanwhile there is a number of non-equilibrium approaches to this problem ranging from the use of phenomenological parameters describing the deviation on the hadronic level (fugacities) [10,11] through linear [12] and nonlinear (constrained) coalescence models [13,14], and mixed quark-hadron chemistry [16] up to very detailed microscopic simulations of the hadronization (parton cascades [17–20] VENUS [21], HIJING [22,23], RQMD [24,25], UrQMD [26–29], string model [30–33], chromodielectric model [34]). The basic physical concepts of these approaches oppose hadronic equilibrium; it might be reached in course of time, but for some components it is actually not. For recent transport simulations at SPS energies see [35]. There are also recent doubts whether chemical equilibrium can be reached at RHIC [36].

The purpose of this letter is to identify a useful quantitative factor in describing the approach to hadronic equilibrium states with quark level processes instead of hadronic ones in a fast expanding fireball. In order to concentrate on this quantifying effort of the time evolution of a complicated system we shall deal with a simple fusion process on the constituent level:



An example is given by the fusion of an s quark and a ud diquark to Λ particle. In this case baryon charge and strangeness conservation constrains the equilibrium.

In such a process the energy-impulse balance is assumed to be satisfied by the surrounding medium, we are interested in the time evolution of the number of final state products $N_C(t)$. The equilibrium number of precursors $N_{A,eq}$ and $N_{B,eq}$ - in a quark matter scenario - will eventually be taken as zero. The initial state is assumed to be far from this equilibrium, it is oversaturated by A and B . The simplest is to consider no C particles at all in the beginning. As the reaction proceeds there will be C type particles at the end (or equilibrium will never be reached), they are hadrons. The cross section for such a reaction - also as a simplification - can be taken as a constant parameter, but temperature and reaction volume must be considered as time dependent quantities. We shall point out, however, that the assumption of constant cross sections actually contradicts to the confinement principle in quark matter

*On leave from KFKI Budapest, Hungary

scenarios, so in that case the hadronization cross section has to be quark density dependent. This extra dependency, not derivable from perturbative or lattice QCD so far, will be referred as “dynamical confinement” in this paper.

In the followings – after a brief presentation of general chemical equations – we shall consider the simple model of hadronization due to coalescence (fusion in the presence of a medium). Comparison of different limiting cases (allowing or excluding the precursors A and B in equilibrium) and estimates of the characteristic deviation from hadronic equilibrium with and without dynamical confinement will close this study.

In the chemical approach to this process all particle numbers are time dependent. The entropy grows during a spontaneous approach towards equilibrium. A general reaction can be characterized by stoichiometric coefficients $\alpha_i^{(r)}$ in a fashion arranged to zero:

$$\sum_i \alpha_i^{(r)} N_i \longleftrightarrow 0$$

for each reaction r . In this arrangement the $\alpha_i^{(r)}$ -s can be negative as well as positive. An important quantity with respect to approaching chemical equilibrium is the activity of a reaction,

$$x^{(r)} = \sum_i \alpha_i^{(r)} \frac{\mu_i}{T}. \quad (1)$$

It describes the (un)balance of gain and loss terms. Rate equations summarize contributions from each reaction channel,

$$\dot{N}_i = \sum_{(r)} \alpha_i^{(r)} R^{(r)} \left(1 - e^{x^{(r)}} \right). \quad (2)$$

Here the factors $R^{(r)}$ depend on numbers of other particles, cross sections and volume. Due to known (e.g. adiabatic) expansion a relation between volume V and temperature T is given.

In simple scenarios of expansion $VT^{3/2} = \text{const.}$ (massive ideal gas expands adiabatically, no back effect from chemistry is taken into account). Now

$$N_i = N_{i,\text{eq}} e^{\mu_i/T} \quad (3)$$

and

$$N_{i,\text{eq}} = V d_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}. \quad (4)$$

In simple scenarios like this the time dependence of particle numbers, $N_i(t)$, is entirely due to that of the $(\mu_i(t) - m_i)/T(t)$. The equilibrium ratio,

$$R_{\text{eq}} = \frac{N_{A,\text{eq}} N_{B,\text{eq}}}{N_{C,\text{eq}}} = \frac{d_A d_B}{d_C} \left(\frac{m_A m_B}{m_C} \right)^{3/2} V T^{3/2} e^{(m_C - m_A - m_B)/T} \quad (5)$$

is nevertheless constant for adiabatic expansion ($VT^{3/2} = \text{const.}$) and composite final products ($m_C = m_A + m_B$).

All rate equations drive the system towards chemical equilibrium. The total entropy production can be obtained from

$$TdS + \mu_i dN_i = dE + pdV = 0 \quad (6)$$

for adiabatic flow of an ideal (“dry”) fluid. One obtains

$$\dot{S} = - \sum_i \frac{\mu_i}{T} \dot{N}_i = \sum_r R^{(r)} x^{(r)} \left(e^{x^{(r)}} - 1 \right) \geq 0. \quad (7)$$

In chemical equilibrium $x^{(r)} = 0$ for each reaction, i.e.

$$\sum_i \alpha_i^{(r)} \mu_i = 0. \quad (8)$$

There are as many linear constraints for the chemical potentials μ_i as many reactions. If this is less than the number of particle sorts, the equilibrium state is undetermined. If this is greater than the number of particle sorts, the reactions cannot be all independent: there are conserved quantities (e.g. from $\dot{N}_1 = \dot{N}_2$ it follows that $B = N_1 - N_2$ is conserved). In this latter case all μ_i can be expressed by a few numbers corresponding to these conserved quantities. This is the constrained equilibrium.

Let us denote the conserved quantities by Q_a ($a = \text{baryon, strangeness, etc.}$). The corresponding charge of the i -th particle sort let be $q_{i,a}$. We have

$$\sum_i q_{i,a} \dot{N}_i = 0 \quad (9)$$

Substituting the rate equations (2) we get

$$\sum_i \sum_r \alpha_i^{(r)} q_{i,a} R^{(r)} \left(1 - e^{x^{(r)}} \right) = 0 \quad (10)$$

Since conservation is fulfilled even out of equilibrium, we conclude that

$$\sum_i \alpha_i^{(r)} q_{i,a} = Q_a^{(r)} = 0, \quad (11)$$

i.e. there is no charge unbalance in any of the reactions. This fact allows us to redefine the chemical potentials as

$$\mu_i = \sum_a q_{i,a} \mu_a + \tilde{\mu}_i. \quad (12)$$

The first contribution, depending on only as many “chemical potentials” as conserved charges we have, does not change the activities:

$$x^{(r)} = \sum_i \sum_a \alpha_i^{(r)} \frac{q_{i,a} \mu_a}{T} + \sum_i \alpha_i^{(r)} \frac{\tilde{\mu}_i}{T}. \quad (13)$$

Since the first part is zero due to eq.(11), chemical equilibrium (vanishing activities) means

$$\sum_i \alpha_i^{(r)} \frac{\tilde{\mu}_i}{T} = 0. \quad (14)$$

For a well-determined or overdetermined system the only equilibrium solution is given by $\tilde{\mu}_i = 0$. In this (constrained) equilibrium state the original chemical potentials are a linear combination of a few μ_a -s, associated to conserved charges,

$$\mu_{i,eq} = \sum_a q_{i,a} \mu_a. \quad (15)$$

Whether this chemical equilibrium state can be reached in a finite time depends on the rates $R^{(r)}$ and activities $x^{(r)}$. In this respect whether a quantity is conserved or not is determined by the set of rate equations. This way it happens that not only real conserved charges but also the number of each constituent quark may be conserved in a sudden hadronization process, like ALCOR assumes this.

For ideal massive particles (Boltzmann distribution)

$$\mu_i = T \log \frac{N_i}{N_{i,eq}}. \quad (16)$$

The activities hence contain ratios of equilibrium numbers in each reaction (mass action law, detailed balance). For a simplified expansion picture these ratios are (nearly) constant. The factors $R^{(r)}$ contain precursor numbers, averaged cross sections (fluxes) and the reaction volume. In an expanding system its time-dependence is far from trivial.

For a general set of reactions and particle sorts the time evolution can only be followed numerically. In order to understand, however, possible deviations from equilibrium in a final state - even after infinitely long time - we consider an oversimplified system in the followings.

In the case of only one reaction channel, $A + B \longleftrightarrow C$, we denote the equilibrium ratio by R_{eq} (cf. eq.(5)). Since $\alpha_A = \alpha_B = 1$ and $\alpha_C = -1$ for this very reaction, we have $\dot{N}_A = \dot{N}_B = -\dot{N}_C$. From this two conservation laws follow,

$$N_A + N_C = N_A(0) = N_0, \quad (17)$$

$$N_B + N_C = N_B(0) = N_0. \quad (18)$$

For considering an evolution towards equilibrium the specific rate,

$$\nu = \frac{\langle \sigma v \rangle}{V}, \quad (19)$$

is a useful quantity. It occurs in the rate equation

$$\dot{N}_C = \nu (N_A N_B - R_{eq} N_C). \quad (20)$$

Substituting the conservation laws (17,18) we arrive at

$$\dot{N}_C = \nu ((N_0 - N_C)(N_0 - N_C) - R_{eq} N_C) = \nu (N_C - N_+)(N_C - N_-). \quad (21)$$

This rate is vanishing for the particle numbers $N_C = N_{\pm}$ with

$$N_{\pm} = R_{eq}/2 + N_0 \pm \sqrt{R_{eq}^2/4 + R_{eq} N_0}. \quad (22)$$

Here N_- is a stable, N_+ is an unstable equilibrium point. For a non-relativistic, adiabatic expansion

$$V T^{3/2} = \text{const.} \quad (23)$$

and hence $R_{eq} = \text{const.}$ for constituent hadrons (those with mass $m_C = m_A + m_B$). A realistic limit is $R_{eq} = 0$ when no A or B particles are allowed in equilibrium. In particular this is the case for quark confinement.

The simple rate equation (21) can be solved analytically giving

$$\int_{N_C(0)}^{N_C(\infty)} \frac{dN_C}{(N_C - N_+)(N_C - N_-)} = \int_{t_0}^{t_{br}} \nu dt = r. \quad (24)$$

Here t_0 is the initial time, when the fireball contains no hadrons, $N_C(0) = 0$. t_{br} is the break-up time, beyond which no chemical reactions (even no particle collisions) occur. In some cases it is large compared to characteristic times and can be approximated by infinity. Now using the identity

$$\frac{1}{(N_C - N_+)(N_C - N_-)} = \frac{1}{N_+ - N_-} \left(\frac{1}{N_C - N_+} - \frac{1}{N_C - N_-} \right) \quad (25)$$

we arrive at

$$\log(N_C - N_+) - \log(N_C - N_-)|_0^{N_C(\infty)} = (N_+ - N_-)r. \quad (26)$$

This leads to the solution

$$N_C(\infty) = N_+ N_- \frac{1 - K}{N_+ - KN_-} \quad (27)$$

with

$$K = \exp(-(N_+ - N_-)r). \quad (28)$$

Here N_- is the constrained equilibrium value, achieved for $K = 0$. There are two alternative forms of this relation derived by using $N_+ N_- = N_0^2$. The first one is the coalescence form,

$$N_C(\infty) = N_0^2 \frac{1 - K}{N_+ - KN_-} = C(K, N_0) N_0^2 = C(K, N_0) N_A(0) N_B(0), \quad (29)$$

the second one is the equilibrium form,

$$N_C(\infty) = N_- \frac{1 - K}{1 - K(N_-/N_0)^2} = \gamma_C N_- = \gamma_C N_{C,\text{eq}}. \quad (30)$$

In particular $N_C(\infty) = N_-$ in the $K = 0$ (infinite rate) case. On the other hand in the case of confinement $R_{eq} = 0$, from which $N_+ = N_- = N_0$ follows.

This agrees with the ALCOR assumption: after long enough time all quarks A and B are converted into hadrons C. This means $N_- = N_0$, satisfied for $R_{eq} = 0$, i.e. when the quark equilibrium numbers are zero (confinement in equilibrium). Then ALCOR ends up with the equilibrium number. (Note: ALCOR does not take into account the gain/loss balance as if it were infinite activity in favor of hadrons - against quarks. This feature is equivalent to putting $R_{eq} = 0$.)

Without quark confinement, but $(N_+ - N_-)$ small ($\mathcal{O}(R_{eq})$), i.e. when precursors are suppressed in the equilibrium state, we consider a negligible, but not zero R_{eq} . In the limit of $R_{eq} \rightarrow 0$ the precursors A and B are suppressed in equilibrium, according to the static confinement principle. This is, however, insufficient for hadronizing all quarks in an expanding matter. We show this in the followings.

We expand K in terms of $(N_+ - N_-)$ which is in the same order of magnitude. We get

$$K = \exp(-(N_+ - N_-)r) = 1 - r(N_+ - N_-) \quad (31)$$

and the final hadron number becomes

$$N_C(\infty) = N_0^2 \frac{r}{1 + rN_0}. \quad (32)$$

Eq. (32) – valid if the precursors are suppressed in chemical equilibrium (and this is all we know from lattice QCD) – has different readings. For r small it comes close to the linear coalescence equation

$$N_C(\infty) = rN_0^2 = rN_A(0)N_B(0). \quad (33)$$

(Note that $N_A(0) = N_B(0) = N_0$.) ALCOR uses extra modification factors b_A and b_B by assuming the modified coalescence rate

$$N_C(\infty) = rb_A b_B N_0^2. \quad (34)$$

To start with it seems to be rather similar to the coalescence picture. Due to conservation, however, ALCOR obtains

$$N_C(\infty) = rb_A b_B N_0^2 = \frac{r}{1 + rN_0} N_0^2 \quad (35)$$

and determines the factors b_i accordingly (depending on N_0). It leads exactly to the constrained equilibrium result in the limit $rN_0 \rightarrow \infty$:

$$N_C(\infty) = N_0. \quad (36)$$

So, reversing the argument, we conclude that for quark matter hadronization r has to diverge in order to reflect confinement in the end stage.

Finally the chemistry approach gives the interpretation of the fugacity due to

$$N_C(\infty) = N_0^2 \frac{r}{1 + rN_0} = \gamma_C N_0. \quad (37)$$

In fact the difference between these approaches is more pronounced by inspecting chemical potentials instead of final numbers. In the discussed simple case the chemical potential of the (final state) hadron is given by

$$\frac{\mu_C}{T} = \log \frac{N_C(\infty)}{N_{C,eq}}. \quad (38)$$

Substituting our result for $N_C(\infty)$ it splits to contributions with different physical meanings:

$$\frac{\mu_C}{T} = \log \frac{N_-}{N_{C,eq}} + \log \frac{N_C(\infty)}{N_-} = \frac{q_{C,1}\mu_1 + q_{C,2}\mu_2}{T} + \log \gamma_C. \quad (39)$$

The conserved quantities (charges) carried by the hadron C are those involved in the identities respected by the rate equations, $N_A(t) + N_C(t) = N_A(0) = N_0$ and $N_B(t) + N_C(t) = N_B(0) = N_0$. In this case $q_{C,1} = 1$ and $q_{C,2} = 1$, respectively. The fugacity parameter (as used by Rafelski et al.) is the remaining part of the formula, in equilibrium it is $\gamma_C = 1$.

Finally we note that the ALCOR approach (constrained coalescence) leads to equilibrium result only if a single reaction channel is considered, like in the case discussed in the present paper. For several, competing channels the relative hadron ratios are different for ALCOR and constrained equilibrium. This best can be seen by inspecting the corresponding μ_i chemical potentials to the pre resonance decay hadron numbers as a function of strangeness in the ALCOR simulation. Slight deviations from the perfect straight lines, which would mark the constrained equilibrium in the ideal case, can be observed [37].

In case of confinement for particles A and B the equilibrium ratio R_{eq} vanishes and N_- has to be replaced by N_0 in the above formula. Exactly this way are usually the chemical potentials associated to conserved charges, μ_1 and μ_2 , determined. The fugacity factor

$$\gamma_C = \frac{1 - K}{1 - K(N_-/N_0)^2}, \quad (40)$$

in only static confining scenarios reduces to

$$\gamma_C = \frac{rN_0}{1 + rN_0} < 1. \quad (41)$$

This must be viewed as an indication for the necessity of considering dynamic confinement in hadronization.

There is no difference between ALCOR and constrained equilibrium in this simple model if confinement is implemented: both lead to $\gamma_C = 1$ eventually. The linear coalescence picture on the other hand assumes $rN_0 \ll 1$, corresponding to a rather impressive deviation from equilibrium. Therefore it is interesting to estimate the factor rN_0 for different heavy-ion experiments.

We continue by estimating the integrated specific rate r . Its value depends not only on cross section(s) – first assumed to be constant and later density dependent – but also on the expansion and cooling of the reaction zone.

We consider here two limiting cases: that of the one dimensional expansion with a rather relativistic equation of state (ideal gas with particles having negligible masses) and that of three dimensional expansion of a massive, non-relativistic ideal Boltzmann gas. The first case is thought to be rather characteristic for RHIC and LHC, the latter rather for *SPS* and *AGS* energies.

During a one-dimensional expansion the volume grows linearly in time, $V = \pi R_0^2 t$, and the (thermal) average of relative velocities is near to the light speed $\langle v \rangle = 1$. In this case we obtain

$$r = \int_{t_0}^{t_{br}} \frac{\sigma}{\pi R_0^2} \frac{dt}{t} = \frac{\langle \sigma v_0 \rangle}{V_0} t_0 \log \frac{t_{br}}{t_0}. \quad (42)$$

The time-integrated specific rate grows with the break-up time t_{br} monotonically, it can eventually be infinite, if there is no thermal break-up of the expanding system. In that case $\gamma_C = 1$, equilibrium is achieved. For a three-dimensional expansion one gets $V = V_0(t/t_0)^3$. Furthermore a massive, non-relativistic ideal gas satisfies $VT^{3/2} = V_0 T_0^{3/2}$ during an adiabatic expansion. The average relative velocity is temperature dependent like $\langle v \rangle \sim \sqrt{T/m} \sim 1/t$, with some reduced mass of the fusing pair m . This together leads to the estimate

$$r = \frac{\langle \sigma v_0 \rangle}{V_0} \int_{t_0}^{t_{br}} t_0^4 \frac{dt}{t^4}. \quad (43)$$

Evaluating the integral we arrive at

$$r = \frac{1}{3} \frac{\langle \sigma v_0 \rangle}{V_0} t_0 \left(1 - \left(\frac{t_0}{t_{br}} \right)^3 \right). \quad (44)$$

It is interesting to note that r is finite even without break-up, i.e. in the $t_{br} \rightarrow \infty$ limit. This also means that the non-relativistic, 3-dimensional spherical expansion scenario with constant hadronization cross section cannot comply with quark confinement. In this case σ has to be time dependent at least as $\sigma \sim t^3 \sim V \sim 1/n$, i.e. the hadronization cross section has to be inversely proportional to the density of quarks. Exactly this assumption was made in transchemistry [16].

In order to simplify the further discussion we relate the thermal average of the rate at the beginning of the expansion to estimates of the collision time due to mean free path,

$$t_c = \frac{1}{\langle \sigma v_0 \rangle \rho_0}, \quad (45)$$

with density $\rho_0 = 2N_0/V_0$. This leads to

$$\frac{\langle \sigma v_0 \rangle}{V_0} = \frac{1}{2N_0 t_c}. \quad (46)$$

The quantity describing the deviation from constrained equilibrium for hadrons then is given as follows:

$$rN_0 = \frac{t_0}{2t_c} \log \frac{t_{br}}{t_0} \quad (47)$$

for one-dimensional relativistic flow and

$$rN_0 = \frac{t_0}{6t_c} \left(1 - \left(\frac{t_0}{t_{br}} \right)^3 \right) \quad (48)$$

for three-dimensional non-relativistic flow of massive constituents. The general dependence of the fugacity on the estimated collision time is in both cases

$$\gamma_C = \frac{1}{1 + \beta(t_c/t_0)}, \quad (49)$$

just the constant β differs. For the relativistic, one-dimensional flow case it is

$$\beta = \frac{2}{\log \frac{t_{br}}{t_0}} \quad (50)$$

and for three-dimensional spherical flow of massive particles it is given by

$$\beta = \frac{6}{1 - \left(\frac{t_0}{t_{br}}\right)^3}. \quad (51)$$

From these formulae one realizes that for $t_{br} = t_0$, i.e. in case of immediate break-up the chemical equilibrium cannot be approached, no hadron C will be formed and γ_C remains zero. The opposite limit, $t_{br} \rightarrow \infty$ is less uniform: while in the relativistic one-dimensional scenario equilibrium is always reached if enough time is given ($\gamma_C(\infty) = 1$), this is not so for a stronger, more spherical expansion. In the latter case γ_C tends to a finite value,

$$\gamma_C(\infty) = \frac{1}{1 + 6t_c/t_0}, \quad (52)$$

which is in general less than one. It means that chemical equilibrium cannot be really reached even in infinite time. (An analysis of proton – deuteron mixture with a similar conclusion was given in Ref. [38].) For early break-up, $t_{br} - t_0 \ll t_0$, both scenarios lead to

$$\beta = \frac{2t_0}{t_{br} - t_0}. \quad (53)$$

In quark matter hadronization scenarios eq.(52) contradicts to the quark confinement principle. In order to reach $\gamma_C(\infty) = 1$ a dynamical confinement mechanism, causing the quark fusion cross section in medium to be (at least) inversely proportional to the quark density, has to be taken into account besides the equilibrium suppression of quark number (referred in this paper as “static confinement”).

Let us assume that A and B are quark level (colored) objects, and the fusion cross section scales as

$$\sigma = \sigma_0 \left(\frac{V}{V_0} \frac{2N_0}{N_A + N_B} \right)^{1+\epsilon} \quad (54)$$

with some extra power ϵ . The solution of the rate equation in the $R_{\text{eq}} = 0$ case (both static and dynamic confinement) becomes

$$N_C = N_0 \left(1 - (1 - \delta)^{1/\epsilon} \right), \quad (55)$$

with

$$\delta = \frac{t_0}{6t_c} \left(\left(\frac{t}{t_0} \right)^{3\epsilon} - 1 \right). \quad (56)$$

For $\epsilon = -1$ (without dynamical confinement) one gets back the result discussed so far. In the limiting case $\epsilon = 0$ (minimal dynamical confinement) one obtains

$$N_C = N_0 \left(1 - \left(\frac{t_0}{t} \right)^{\frac{t_0}{2t_c}} \right). \quad (57)$$

Fig.1 plots the dependence of the non-equilibrium fugacity factor γ_C on the scaled break-up time t_{br}/t_0 for these scenarios: one and three dimensional scaling expansion, in the three-dimensional case both for static and dynamic confinement (color density dependent quark fusion cross section). The uppermost curve, showing the fastest equilibration

belongs to a 3-dimensional expansion scenario with a cross section proportional to the inverse of the quark density (cf. eq.(57)). The lowest curve, saturating well below the equilibrium value, corresponds to the same expansion with static confinement only. The middle curve shows the static RHIC scenario, reaching equilibrium (vanishing quark number) eventually, but quite slowly. For all calculations we used the ratio $t_0/t_c = 3$ for the sake of simplicity. (The smaller t_0 , the higher density, therefore the shorter inter-collision time t_c initially.)

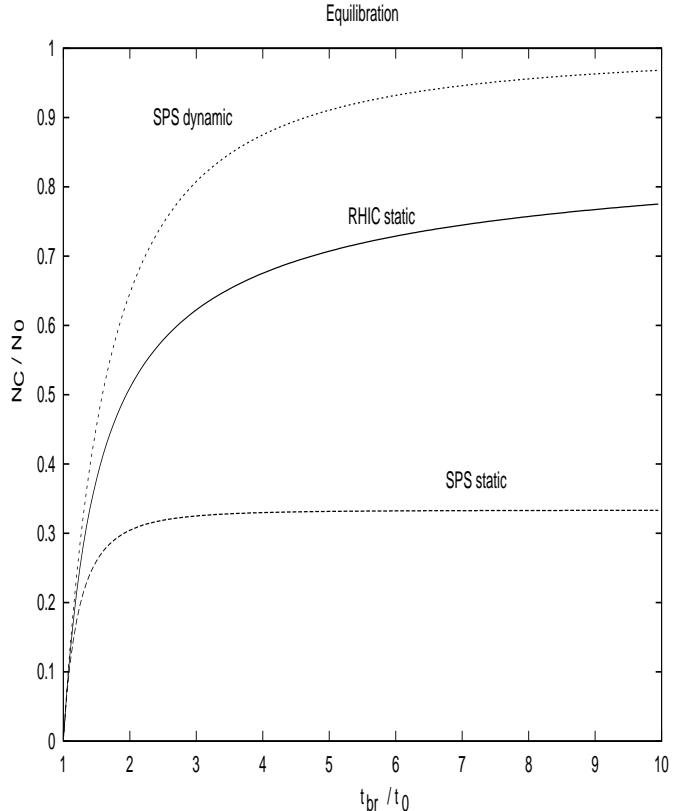


FIG. 1. Degree of the chemical equilibrium reached for different break-up times t_{br}/t_0 and initial inter-collision times t_c/t_0 for SPS and RHIC expansion scenarios with static and dynamic confinement.

Finally we compare characteristic values for SPS and RHIC. In the SPS scenarios we assume $t_0 = 1 \text{ fm}/c$ as the starting time of the expansion. The collision time changes somewhat due to different temperatures (and effective quark masses) presumed at the beginning of hadronization. We consider a value characteristic for quark-gluon plasma, $t_c \approx 0.33 \text{ fm}/c$. We take over the ratio $t_0/t_c = 3$ for the RHIC scenario, too. The break-up time is the most unknown factor, therefore the equilibration has been plotted as function of t_{br}/t_0 . Fortunately, once the saturation value of γ_C has been approached, there is no strong dependence on it. The $1 - e^{-1} \approx 0.63$ fraction of equilibrium value is reached after $2 \text{ fm}/c$ for the dynamic SPS, after $3.5 \text{ fm}/c$ for the static RHIC scenario. In the static SPS scenario it will be never reached, γ_C saturates near 0.3 already after $2 \text{ fm}/c$. This last scenario, quark coalescence with density independent probability, can be excluded on the basis of quark confinement.

In conclusion 1) the results of the statistical model can be interpreted not only by assuming hadronic equilibrium, but as well by quark – hadron mixture equilibrium with quarks eventually suppressed due to confinement. This interpretation avoids the collision time problem: the hadrons need not transmute into each other directly. 2) The ALCOR and constrained hadronic equilibrium leads to the same result in this simple scenario due to too many constraints on conserved quantities. Quark coalescence, which takes conservation into account, therefore mimics hadrons in chemical equilibrium. This feature certainly does not generalize for several competing reaction channels, but the overall picture can be similar. ALCOR actually assumes more conserved quantities, than usual: quark and

antiquark numbers in any flavor are conserved separately due to the assumption of rapid hadronization. 3) $\gamma_C \approx 0.3$ is typical for CERN SPS without considering dynamical confinement, even for an arbitrary long hadronization process. This leads to a factor two to three characteristic error on equilibrium model results. The experimental evidence, reaching at least 70-80% of equilibrium ratios for almost all hadrons in a short time in the order of several fm/c, excludes this possibility. It also underlines the quark matter hadronization scenario with dynamical confinement effect, as transchemistry describes it. 4) An expansion scenario typical for RHIC and LHC energies in principle allows for reaching the equilibrium even with static confinement, but the process is rather slow and an early break-up may prevent from hadronizing all quarks. Dynamical confinement has to be in work for one-dimensional expansion scenarios, too.

One is tempted to speculate about the origin of dynamical confinement, looking for a cause of the density dependence of the hadronization cross section. Since this dependence makes the cross section stronger with diminishing quark density, it cannot have perturbative origin; on the contrary it has to do with formation of strings or color ropes. As the color density drops, strings, follow-ups the chromoelectric flux lines, become longer before ending on a partner color. The volume, in which possible partner to form a hadron occur, hence increases, so does the hadronization cross section, $\sigma \sim \ell^3$. The characteristic string length on the other hand depends on the color density as $\sim n_{\text{color}}^{-1/3}$. Such a mechanism would lead to $\sigma = \sigma_0 n_{\text{color}}(0)/n_{\text{color}}$.

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- [1] J. Rafelski and B. Müller, “Strangeness Production In The Quark - Gluon Plasma,” *Phys. Rev. Lett.* **48** (1982) 1066.
- [2] T. Biró and J. Zimányi, “Quarkochemistry In Relativistic Heavy Ion Collisions,” *Phys. Lett.* **B113** (1982) 6.
- [3] T. S. Biró and J. Zimányi, “Quark Gluon Plasma Formation In Heavy Ion Collisions And Quarkochemistry,” *Nucl. Phys. A395* (1983) 525.
- [4] P. Koch, B. Müller and J. Rafelski, “Strangeness In Relativistic Heavy Ion Collisions,” *Phys. Rept.* **142** (1986) 167.
- [5] P. Braun-Münzinger, I. Heppe and J. Stachel, “Chemical equilibration in Pb + Pb collisions at the SPS,” *Phys. Lett. B465* (1999) 15 [[nucl-th/9903010](#)].
- [6] F. Becattini, M. Gazdzicki and J. Sollfrank, “Thermal fits of hadron abundances from p p to A A collisions,” *Nucl. Phys. A638* (1998) 403.
- [7] F. Becattini, M. Gazdzicki and J. Sollfrank, “On chemical equilibrium in nuclear collisions,” *Eur. Phys. J. C5* (1998) 143 [[hep-ph/9710529](#)].
- [8] J. Cleymans, H. Oeschler and K. Redlich, “Particle ratios at SPS, AGS and SIS,” *J. Phys. G G25* (1999) 281 [[nucl-th/9809031](#)].
- [9] N. J. Davidson, H. G. Miller, D. W. von Oertzen and K. Redlich, “Thermal gas models and particle production in heavy ion collisions,” *Z. Phys. C56* (1992) 319.
- [10] J. Rafelski, J. Letessier and A. Tounsi, “Thermal strangeness and charm in QGP,” *Heavy Ion Phys.* **4** (1996) 181.
- [11] J. Letessier and J. Rafelski, “Chemical non-equilibrium and deconfinement in 200-A-GeV sulphur induced reactions,” *Phys. Rev. C59* (1999) 947 [[hep-ph/9806386](#)].
- [12] A. Bialas, “Quark model and strange baryon production in heavy ion collisions,” *Phys. Lett. B442* (1998) 449 [[hep-ph/9808434](#)].
- [13] T. S. Biró, P. Lévai and J. Zimányi, “ALCOR: A Dynamic model for hadronization,” *Phys. Lett. B347* (1995) 6.
- [14] J. Zimányi, T. S. Biró and P. Lévai, “The dependence of strange hadron multiplicities on the speed of hadronization,” *J. Phys. G G23* (1997) 1941 [[hep-ph/9712352](#)].
- [15] J. Zimányi, T. S. Biró and P. Lévai, “The mischievous linear coalescence model and the correct quark counting in heavy ion collisions,” [hep-ph/9904501](#).
- [16] T. S. Biró, P. Lévai and J. Zimányi, “Hadronization with a confining equation of state,” *Phys. Rev. C59* (1999) 1574.
- [17] K. Geiger and B. Müller, “Parton cascades in highly relativistic nuclear collisions,” *Nucl. Phys. A544* (1992) 467C.
- [18] K. Geiger, “Particle production in high-energy nuclear collisions: A Parton cascade cluster hadronization model,” *Phys. Rev. D47* (1993) 133.

- [19] K. Geiger, “Non-equilibrium QCD: Interplay of hard and soft dynamics in high-energy multi-gluon beams,” *Phys. Rev. D* **56** (1997) 2665 [hep-ph/9611400].
- [20] K. Geiger, R. Longacre and D. K. Srivastava, “VNI version 4.1: Simulation of high-energy particle collisions in QCD: Space-time evolution of $e^+ e^- \rightarrow A + B$ collisions with parton cascades, parton hadron conversion, final-state hadron cascades,” *nucl-th/9806102*.
- [21] K. Werner, J. Hufner, M. Kutschera and O. Nachtmann, “Particle Production In Hadron Nucleus Collisions At 100-Gev From A Parton String Model,” *Z. Phys. C* **37** (1987) 57.
- [22] K. J. Eskola, B. Müller and X. Wang, “Screening of initial parton production in ultrarelativistic heavy-ion collisions,” *Phys. Lett. B* **374** (1996) 20 [hep-ph/9509285].
- [23] X. Wang, “Dynamical screening and radiative parton energy loss in a quark-gluon plasma,” *nucl-th/0003033*.
- [24] H. Sorge, “Xi and Anti-xi production in central S (200-A-GeV) on W collisions from RQMD,” *Phys. Lett. B* **344** (1995) 35.
- [25] J. Sollfrank, U. Heinz, H. Sorge and N. Xu, “Thermal analysis of hadron multiplicities from RQMD,” *J. Phys. G* **25** (1999) 363 [nucl-th/9811011].
- [26] F. Antinori *et al.* [WA97 Collaboration], “Strangeness enhancement at midrapidity in Pb Pb collisions at 158-A-GeV/c: A comparison with VENUS and RQMD models,” *Eur. Phys. J. C* **11** (1999) 79.
- [27] S. A. Bass *et al.*, “URQMD: A new molecular dynamics model from GANIL to CERN energies,” *Prepared for International Conference on Nuclear Physics at the Turn of Millennium: Structure of Vacuum and Elementary Matter, Wilderness / George, South Africa, 10-16 Mar 1996*.
- [28] S. A. Bass, M. Gyulassy, H. Stöcker and W. Greiner, “Signatures of quark-gluon-plasma formation in high energy heavy-ion collisions: A critical review,” *J. Phys. G* **25** (1999) R1 [hep-ph/9810281].
- [29] M. Bleicher, S. A. Bass, H. Stöcker and W. Greiner, “RHIC predictions from the UrQMD model,” *hep-ph/9906398*.
- [30] N. S. Amelin and L. V. Bravina, “The Monte Carlo Realization Of Quark - Gluon String Model For Description Of High-Energy Hadron-Hadron Interactions,” *Sov. J. Nucl. Phys.* **51** (1990) 133.
- [31] D. J. Dean, M. Gyulassy, B. Müller, E. A. Remler, M. R. Strayer, A. S. Umar and J. S. Wu, “Multiparticle production in lepton nucleus collisions and relativistic string models,” *Phys. Rev. C* **46** (1992) 2066.
- [32] S. A. Bass *et al.*, “Microscopic models for ultrarelativistic heavy ion collisions,” *Prog. Part. Nucl. Phys.* **41** (1998) 225 [nucl-th/9803035].
- [33] P. Csizmadia, P. Lévai, S. E. Vance, T. S. Biró, M. Gyulassy and J. Zimányi, “Strange hyperon and antihyperon production from quark and string-rope matter,” *J. Phys. G* **25** (1999) 321 [hep-ph/9809456].
- [34] C. T. Traxler, U. Mosel and T. S. Biró, “Hadronization of a quark-gluon plasma in the chromodielectric model,” *Phys. Rev. C* **59** (1999) 1620 [hep-ph/9808298].
- [35] E. L. Bratkovskaya, W. Cassing, C. Greiner, M. Effenberger, U. Mosel and A. Sibirtsev, “Aspects of thermal and chemical equilibration of hadronic matter,” *nucl-th/0001008*.
- [36] S. J. Lindenbaum and R. S. Longacre, “Possible striking signals for a quark-gluon plasma at RHIC,” *nucl-th/0004019*.
- [37] T. S. Biró, P. Lévai, J. Zimányi, “Strange Hadrons from Quark and Hadron Matter: a Comparison”, contribution to S’2000 Conference, Berkeley, July 20-25, 2000.
- [38] T. S. Biró, “Asymptotic Hadrochemistry,” *Phys. Lett. B* **143** (1984) 50.